South Australian
Certificate of Education

Essential Mathematics
2023

Question booklet

Topic 2: Measurement (Questions 1 to 3) 30 marks

Topic 4: Statistics (Questions 4 to 6) 30 marks

Topic 5: Investments and loans (Questions 7 to 9) 30 marks

- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on pages 12, 21, and 29 if you need more space
- Allow approximately 40 minutes for ***each*** topic

Examination information

Materials
- Question booklet
- SACE registration number label

Instructions
- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used — complete the box below

Total time: 130 minutes
Total marks: 90

Attach your SACE registration number label here	**Graphics calculator** 1. Brand _____ Model _____ 2. Brand _____ Model _____

Government of South Australia

Question 1 (6 marks)

Diagram 1 below shows some measurements of a block of land.

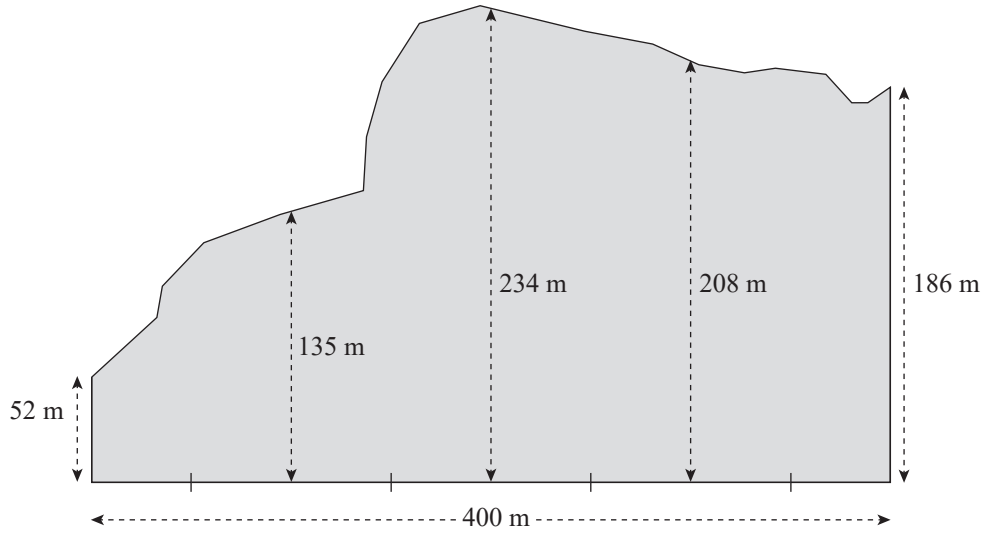


Diagram 1 *[This diagram is not drawn to scale.]*

(a) Using Simpson's rule, calculate the approximate area of the block of land in Diagram 1.

(3 marks)

(b) The area of the block of land can also be approximated using a trapezium.

The dimensions of one possible trapezium are shown in Diagram 2.

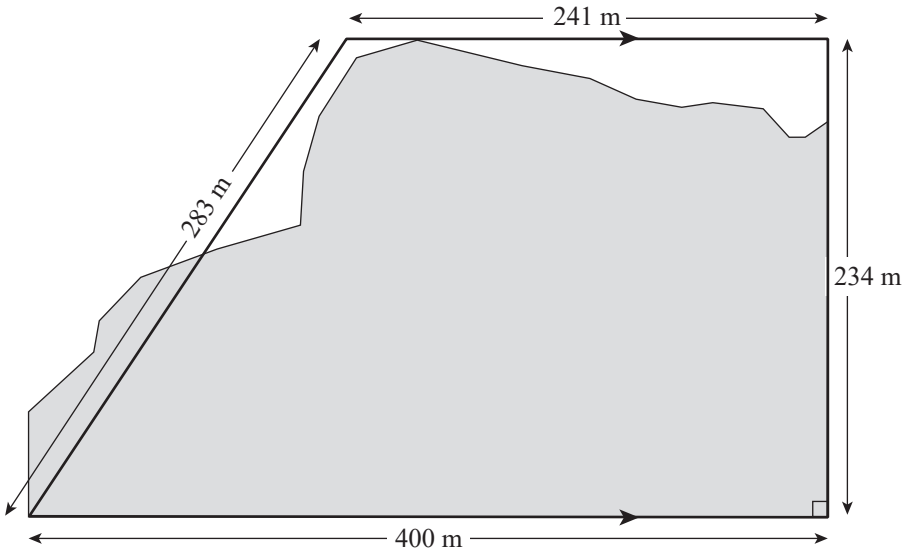


Diagram 2 *[This diagram is not drawn to scale.]*

(i) Calculate the approximate area of the block of land, using the dimensions of the trapezium in Diagram 2.

(1 mark)

(ii) Explain whether or not the trapezium shown in Diagram 2 is a suitable approximation for the area of the block of land.

(2 marks)

Question 2 (10 marks)

When designing a kitchen, there are two basic design rules to consider for positioning the sink, the refrigerator, and the stove cooktop. These design rules are based on a triangle that is drawn connecting these three kitchen items.

The two design rules are:

- Each side of the triangle connecting the three kitchen items should be between 1.25 m and 2.75 m.
- The perimeter of the triangle connecting the three kitchen items should be between 3.6 m and 8.0 m.

Diagram 3 shown below is one design that is being considered for the kitchen.

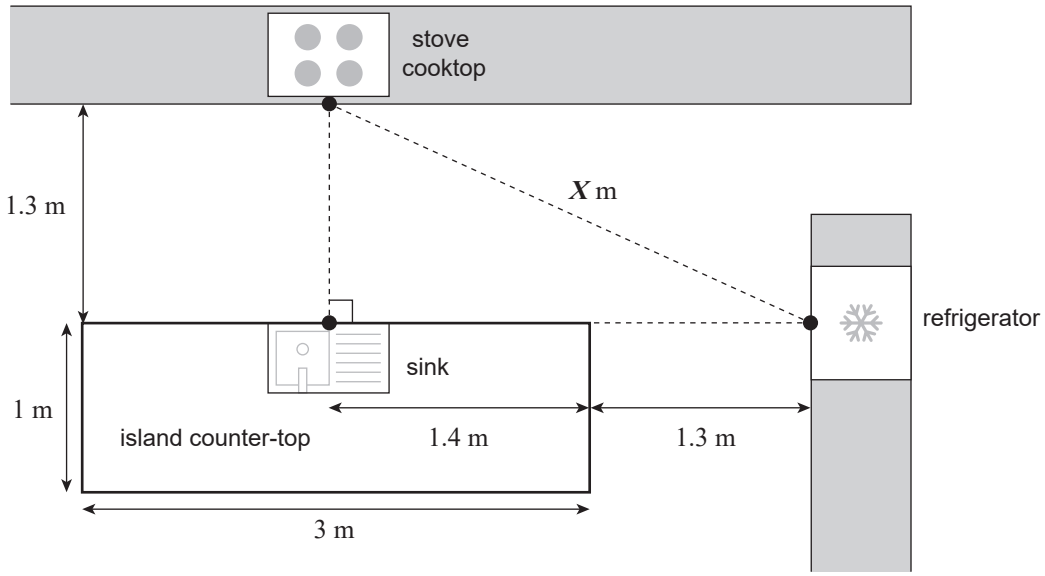


Diagram 3

(a) Using the information in Diagram 3:

(i) Calculate the distance (X) between the stove cooktop and refrigerator for the kitchen design.

(2 marks)

(ii) Calculate the perimeter of the triangle connecting the three kitchen items.

(1 mark)

(iii) Place a tick in the box next to the correct statement below.

- ☐ The position of the sink, refrigerator, and stove cooktop in this kitchen design would satisfy ***both*** design rules.
- ☐ The position of the sink, refrigerator, and stove cooktop in this kitchen design would satisfy ***only one*** of the design rules.
- ☐ The position of the sink, refrigerator, and stove cooktop in this kitchen design would satisfy ***neither*** of the design rules.

(1 mark)

Question 2 continues on page 6.

Another design is shown below in Diagram 4.

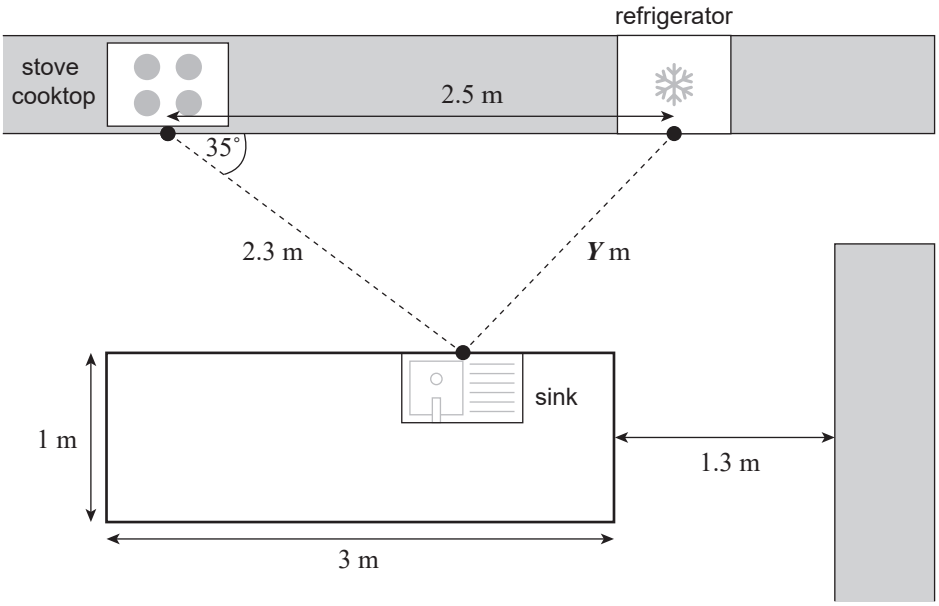


Diagram 4

(b) Using the information in Diagram 4:

(i) Calculate the distance (**Y**) between the sink and refrigerator for the kitchen design, using the cosine rule.

(2 marks)

(ii) Calculate the perimeter of the triangle connecting the three kitchen items.

(1 mark)

For the final kitchen design:

- the sink and stove cooktop are located in the same position as in Diagram 4.
- the distance between the sink and refrigerator is increased to 1.75 m as shown below in Diagram 5.

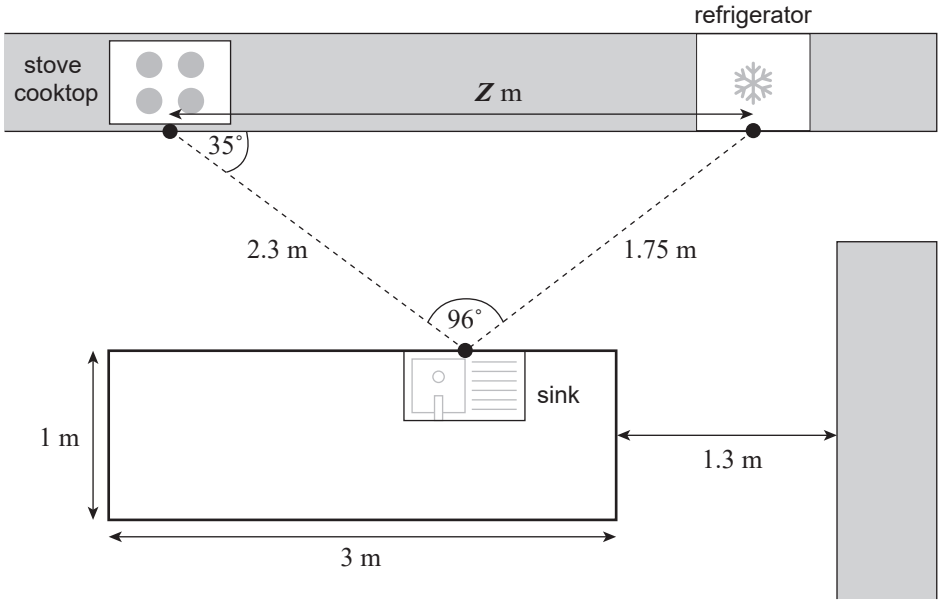


Diagram 5

(c) Using the information in Diagram 5:

(i) Calculate the distance ($\mathbf{Z}$) between the stove cooktop and refrigerator for this kitchen design.

(2 marks)

(ii) State *one* problem with this design, in relation to the design rules.

(1 mark)

Question 3 (14 marks)

Some students are considering collecting soft-drink cans for recycling at their school. The school purchases one recycling bin that has a hemisphere on top of a cube, as shown in Diagram 6.

Image of the recycling bin

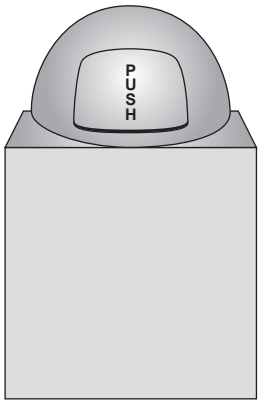


Diagram 6

Dimensions of the recycling bin

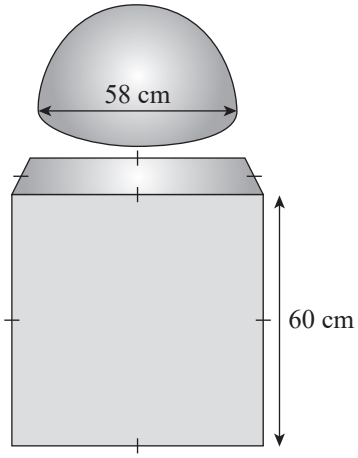


Diagram 7

(a) (i) The cube section of the recycling bin has sides of length 60 cm.

Calculate the volume of the cube.

(1 mark)

(ii) The hemisphere section of the recycling bin has a diameter of 58 cm.

Calculate the volume of the hemisphere.

Hint: Volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

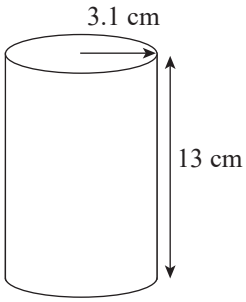
(3 marks)

(iii) Show that the total volume of the recycling bin is approximately $267\,080\ \text{cm}^3$.

(1 mark)

(b) A typical soft-drink can is a cylinder with a radius of $3.1\ \text{cm}$ and a height of $13\ \text{cm}$.

(i) Show that the volume of one typical soft-drink can is approximately $392\ \text{cm}^3$.



(1 mark)

(ii) The students calculated that the recycling bin would hold approximately 680 typical soft-drink cans.

Show a calculation that supports this claim.

(1 mark)

Question 3 continues on page 10.

(iii) Explain whether or not it is reasonable for this number of typical soft-drink cans to fit in the recycling bin.

(2 marks)

The soft-drink cans can be crushed before being put in the bin.

The volume of one crushed soft-drink can is reduced to $136\ \text{cm}^3$.

(c) Calculate the height of the soft-drink can after it has been crushed to a volume of $136\ \text{cm}^3$. Assume that the radius of the soft-drink can remains $3.1\ \text{cm}$.

(2 marks)

Additional recycling bins will be purchased by the school using the money earned from collecting soft-drink cans within one school year.

Each recycling bin costs $\$284$.

On average, 36 soft-drink cans are collected *each school day* and each can is worth 10 cents.

Assume there are 40 weeks in a school year and there are no public holidays during these weeks.

(d) Determine how many recycling bins could be purchased within one school year using only the money earned from collecting the soft-drink cans at school.

Show calculations to support your answer.

(3 marks)

You may write on this page if you need more space to finish your answers to any questions in Topic 2. Make sure to label each answer carefully (e.g. 3(a)(i) continued).

Question 4 (7 marks)

A researcher read an article that claimed 'Public schools are charging more in their canteens than Independent and Catholic schools'. They decided to investigate the claim.

In a state, there are 110 Public, 94 Independent, and 36 Catholic schools.

(a) (i) Calculate the number of schools to be included if the researcher wants to conduct a census.

(1 mark)

(ii) State *two* reasons why the researcher may choose to use a sample rather than conduct a census.

(2 marks)

The researcher decided that it was appropriate to select 30 schools to be included in a sample.

(b) (i) Discuss the reliability of the results if the researcher chooses 10 Public, 10 Independent, and 10 Catholic schools to be included in the sample.

(2 marks)

(ii) Calculate how many Public schools would be included in a stratified sample of 30 schools.

(2 marks)

Question 5 (11 marks)

In some supermarkets the entrance is on the left of the checkouts, whereas in others the entrance is on the right, as shown in Diagrams 8 and 9.

Supermarket A
entrance is left of the checkouts

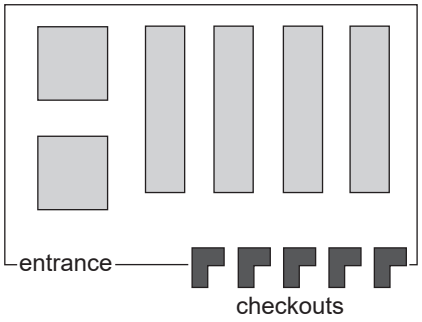


Diagram 8

Supermarket B
entrance is right of the checkouts

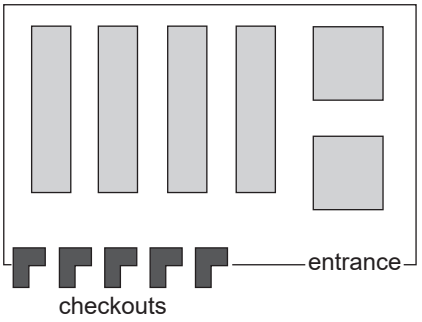


Diagram 9

It is thought that the amount of money spent within the supermarket depends upon whether the entrance is on the left or on the right of the checkouts.

To explore this, some students collected data from two different supermarkets. The students recorded the amount of money spent by 10 customers in each supermarket.

Table 1: Money spent at supermarket ($)

Supermarket A (entrance on left of checkouts)	*Supermarket B (entrance on right of checkouts)*
147	148
78	175
76	112
162	66
76	144
184	189
211	150
220	86
89	110
58	138

Question 5 continues on page 16.

(a) *Complete* Table 2 (correct to one decimal place).

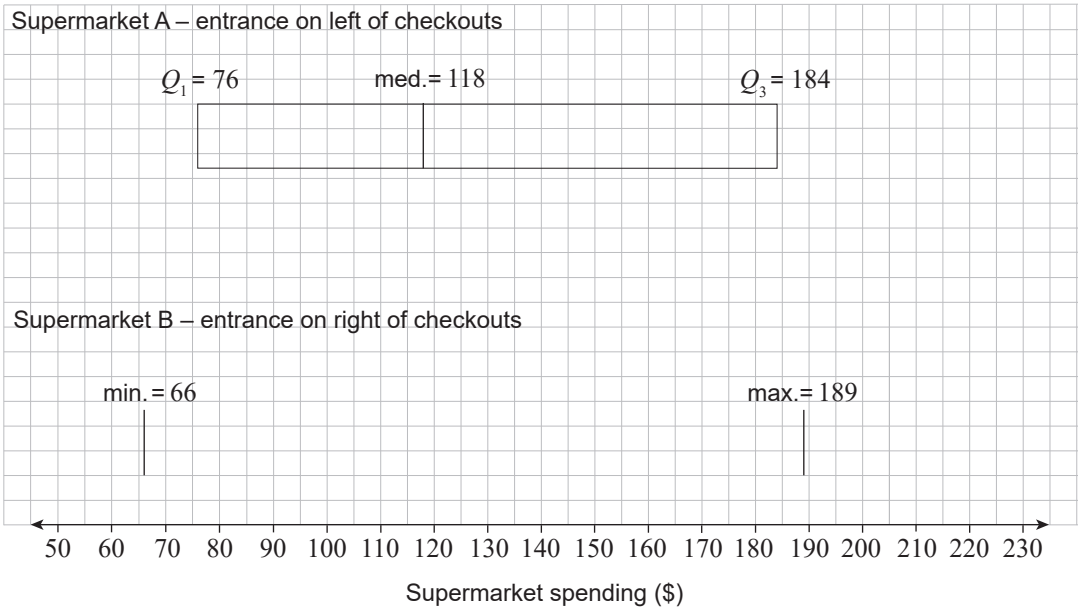
Table 2: Statistical measures of supermarket spending ($)

Statistical measure	*Supermarket A (entrance on left of checkouts)*	*Supermarket B (entrance on right of checkouts)*
mean	130.1	
standard deviation	61.7	
lower quartile (Q_1)	76.0	
median	118.0	141.0
upper quartile (Q_3)	184.0	

(3 marks)

(b) *Complete and label* the box and whisker diagrams below.

Supermarket spending ($)



(3 marks)

(c) Explain whether there is more variability in spending when customers enter the supermarket on the left or on the right of the checkouts. Justify your answer using *one* statistical measure.

(2 marks)

A website claims that a study found 'shoppers entering a supermarket on the right of the checkout, on average, spend approximately $2 more per visit'.

(d) Explain whether or not the claim is supported by the data collected by the students, using *one* statistical measure.

(2 marks)

(e) State *one* aspect (other than which side of the checkouts the customers enter from) that could affect the amount spent by customers in supermarkets.

(1 mark)

Question 6 (12 marks)

Sophie believes that her phone battery is going flat quicker than it should.

She fully charged her phone overnight, and then checked the percentage of charge remaining at different times during the day.

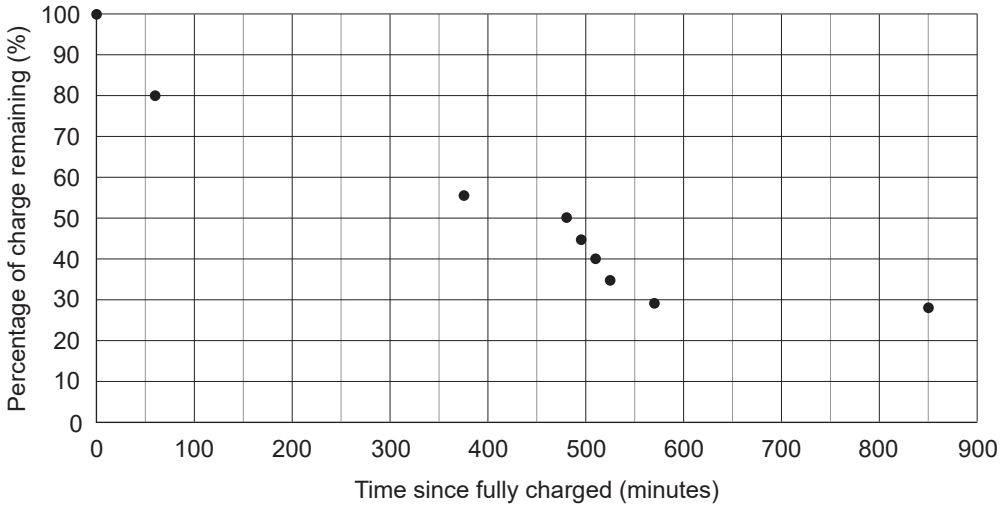
Table 3 below shows the data that Sophie recorded.

Table 3: Data of phone charge

Data collection number	*Time since fully charged (minutes)*	*Percentage of charge remaining (%)*
1	0	100
2	60	80
3	345	65
4	375	55
5	480	50
6	495	45
7	510	40
8	525	35
9	570	29
10	850	28

(a) Plot the data point for Data Collection number 3 on the scatter plot below. (1 mark)

Phone charge data scatter plot



Sophie's day is made up of four periods of time: before school, at school, after school, and at her part-time job after school.

Sophie's school only allows students to access their phones at lunchtimes.

(b) Using the phone charge data from Table 3 and/or the scatter plot:

(i) Complete the statement below.

Sophie finished school at _____ minutes and started her part-time job at _____ minutes.

(2 marks)

(ii) Suggest a reason why the percentage of charge remaining did not decrease significantly between Data Collection numbers 9 and 10.

(1 mark)

(c) Calculate the coefficient of determination (r^2) and state the strength of the relationship between the percentage of charge remaining and the time since fully charged.

(2 marks)

Question 6 continues on page 20.

(d) (i) Determine the equation of the least squares regression line (line of best fit).

(2 marks)

(ii) Using the equation of the least squares regression line, predict the amount of charge remaining on Sophie's phone if she had been allowed to check it during recess time, 165 minutes after the phone was fully charged.

(2 marks)

When Sophie purchased the phone, the website advertised '*A battery that lasts all day*'.

(iii) Using the equation of the least squares regression line, show whether or not the battery is likely to *last all day*.

(2 marks)

You may write on this page if you need more space to finish your answers to any questions in Topic 4. Make sure to label each answer carefully (e.g. 6(b)(ii) continued).

Question 7 (7 marks)

Marni invests $\$12\,000$ for 3 years in an account with a flat rate of 3.85% per annum.

(a) (i) Calculate the interest that Marni will earn over the 3 years.

(1 mark)

(ii) Calculate the *total amount* of money that Marni can withdraw at the end of the 3 years.

(1 mark)

(b) If Marni was able to invest her $\$12\,000$ in an account with the same rate of 3.85% per annum, but compounded monthly, calculate the interest earned over the 3 years.

(3 marks)

Marni realised that the compound rate offered by the bank was only 3.13% per annum, compounded monthly.

(c) Calculate the monthly deposit that is required to achieve the same balance as the original account in part (a)(ii). Assume the investment amount and time remain the same.

(2 marks)

Question 8 (10 marks)

A loan of $\$460\,000$ with monthly repayments for 30 years has an interest rate of 5.08% per annum, compounded monthly.

(a) (i) Show that the minimum monthly repayments would be approximately $\$2500$.

(2 marks)

(ii) Calculate the *total interest* that would be paid through making the minimum monthly repayments of approximately $\$2500$.

(1 mark)

Eleni and Jacob need to borrow $\$460\,000$ to purchase a new house. They are both working full time so have determined that they can afford monthly repayments of $\$3300$, with the interest rate of 5.08% per annum, compounded monthly.

They plan to make these monthly payments for the first 8 years of the loan.

(b) Show that after 8 years of monthly repayments of $\$3300$, Eleni and Jacob would still have a loan balance of approximately $\$300\,200$.

(2 marks)

After the first 8 years, Eleni and Jacob reduce their monthly repayments to the minimum requirement of approximately $\$2500$.

(c) (i) Calculate how long (in months) it would take Eleni and Jacob to repay the remaining loan balance of approximately $\$300\,200$ through making the minimum monthly repayments of $\$2500$. Assume the interest rate remains the same.

(2 marks)

(ii) Calculate the *total interest* paid by Eleni and Jacob.

(2 marks)

(iii) Calculate the interest saved by Eleni and Jacob by making the higher repayments at the start of their loan, when compared to making the minimum monthly repayments for 30 years.

(1 mark)

Question 9 (13 marks)

Oscar is considering three superannuation fund options, as shown below in Table 4.

Table 4: Superannuation fund options

Fund type	*Average rate of return (per annum)*	*Compounding period*
Balanced fund	7.74%	Quarterly
High growth fund	9.15%	Quarterly
Socially responsible fund	7.31%	Quarterly

Each of the returns quoted represent the average return for the previous 10 years.

(a) Quarterly contributions of \$1550 are paid into the *Socially responsible fund* for Oscar for 40 years. Show that Oscar will have a superannuation balance of approximately \$1 453 000.

(2 marks)

(b) Calculate *how much less* would need to be contributed each quarter for Oscar to achieve the same balance after 40 years in the *High growth fund*.

(2 marks)

Oscar receives advice that when you near retirement you should choose lower risk investments to preserve your investment.

Oscar decides to use the *Balanced fund* for the remaining 5 years of his working life.

Assume that after 40 years, Oscar's superannuation balance is $\$1\,453\,000$.

(c) Calculate the superannuation balance if the quarterly contributions of $\$1550$ continue for Oscar in the *Balanced fund* for the next 5 years.

(2 marks)

(d) (i) State *one* assumption that has been made for these superannuation calculations.

(1 mark)

(ii) Explain the impact on Oscar's final superannuation balance if the assumption in part (d)(i) is changed.

(2 marks)

Question 9 continues on page 28.

At retirement, Oscar decides to roll $\$2\,160\,000$ of his superannuation fund balance into a savings account that compounds weekly. He wants the money to last 20 years.

In planning for retirement, Oscar estimates that a withdrawal of $\$2475$ each week will be required for a comfortable lifestyle.

(e) Calculate the interest rate (compounded weekly) that the savings account will need to earn if Oscar withdraws $\$2475$ each week.

(2 marks)

Oscar's withdrawal of $\$2475$ each week during retirement was estimated on the current standard of living.

Inflation is predicted to be 4.3% per annum for the 20 years of Oscar's retirement.

(f) Calculate how much Oscar will need to withdraw in the 20th year if he wants to maintain his lifestyle.

(2 marks)

You may write on this page if you need more space to finish your answers to questions on Topic 5. Make sure to label each answer carefully (e.g.9(d)(ii) continued).