Performance Standards for Stage 1 General Mathematics

Overall grade A

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Formation and testing of appropriate predictions, using sound mathematical evidence.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Attempted development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Formation and testing of mostly appropriate predictions, using some mathematical evidence.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in different contexts. Application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Formation of an appropriate prediction and some attempt to test it using mathematical evidence.
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in context. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted formation of a prediction with limited attempt to test it using mathematical evidence.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to form or test a prediction.

Mini-Box of Chocolates: Measurement Assignment

Stage 1 General Mathematics

Assessment Type 2: Mathematical Investigation

Contents:

Introduction:

In this task, 3 pieces of mathematical-shaped chocolate were constructed – one of a simple 3D form and two, more complex shapes. All chocolates include both planar and curved surfaces. In this assignment, it was beneficial to consider both the profitability and reduction of waste when exploring the packaging and design of chocolates.

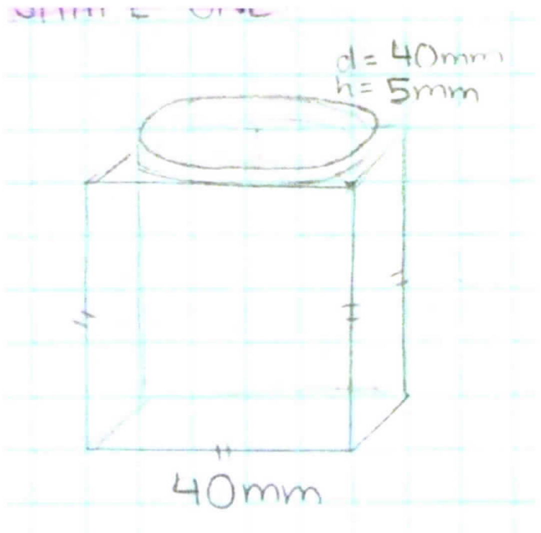
In part one, *'The Design'* sketches of the 3 chocolates with approximate measurements were created. *'Estimating and Calculating Volume'*, part two, volumes for all pieces were estimated and calculated. The results were used to calculate the absolute error and the percentage error. In part three *'Calculating Surface Area'*, the net of each 'slot' that the chocolates sit in in the tray were drawn. A net for the box was also drawn to assist in calculating the total surface area of the tray as well as the total surface area of the cardboard packaging. *'Calculating Cost'* (part four) displays the cost of three parts – the plastic tray, cardboard packaging and the chocolate itself.

To summarise, all findings are displayed in part 5 *'The Report'*. This discusses improvements that could be made to minimise errors, increase accuracy and improve design.

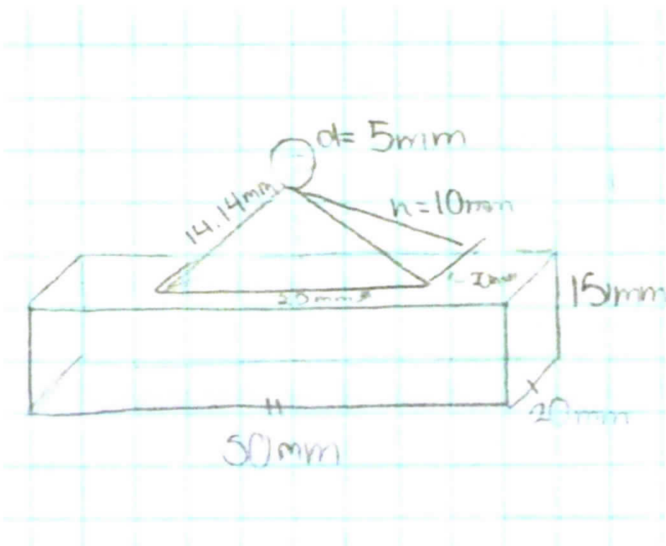
Part One: The Design

Note: These 3 designs also display the accurate measurements of the chocolates, not estimate

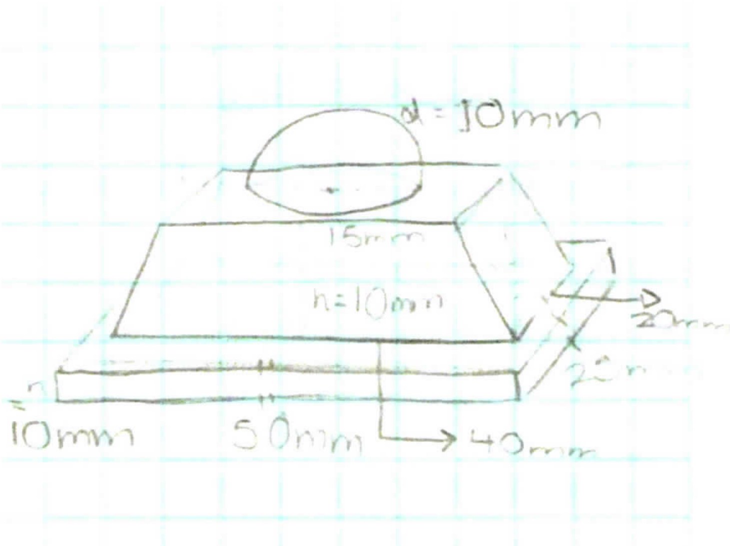
Chocolate Piece Number ONE:



Chocolate Piece Number TWO:



Chocolate Piece Number THREE:



- All complex shapes under 50mm in height chosen.

Part Two: Estimating and Calculating Volume

A. Estimate of volumes:

Piece Number ONE: Estimate values are...

Cube: A length, width and depth of 45mm, using the formula V= L^3
(45^3)
Therefore, the volume of the Cube is 91,125mm^3

For an estimation rather then just increasing dimensions you should have put a cube around the whole shape e g 40x40x45 Something similar could have been calculated for shape 2 & 3.

Cylinder: A height of 5mm and a diameter of 45mm, using the formula V=$\pi r^2 h$,
(πx22.5^2x5)
Therefore, the volume of the Cylinder is 7,952.16mm^3

The estimate volume of piece one is 91,125 + 7,952.16 = 99,077.16mm^3

Piece Number TWO: Estimate values are...

Sphere: A diameter of 5mm, using the formula V= 4/3πr^3
(4/3$\pi 5^3$)
Therefore, the volume of the sphere is 65.45mm^3

Square-Based Pyramid: Abase length of 20mm, a height of 10mm and a slant height of 15mm. Using the formula V=a^2h/3
(20^2x10/3)
Therefore, total volume of square-based pyramid is 1332mm^3

Rectangular Prism: A length of 50mm, a height of 20mm and width of 20mm, using the formula V=lwh,
(50x20x20)
Therefore, the volume of the rectangular prism is 20,000mm^3

The estimate volume of piece two is 65.45 + 1332 + 20,000 = 21,397.45mm^3

Piece Number THREE: Estimate values are...

Semi-sphere: A diameter of 10mm. Using the formula V= $\frac{\left(\frac{4}{3}\pi r\right)}{2}$
$\frac{\left(\frac{4}{3}\pi 10^{3}\right)}{2}$, Therefore, the total volume of the semi-sphere is 261.8mm^3.

Trapezoid: A height of 10mm, a base length of 40mm and a width of 20mm. Using the formula, V= w[1/2h(b1+b2)],
20[1/2 x 10(40+20)]
Therefore, the volume of the trapezoid is 5,500m^3

Rectangular Prism: A length of 50mm, width of 30mm and a height of 10mm. Using the formula V=lwh,
50 x 30 x 10
Therefore, the volume of the rectangular prism is 15,000mm^3

The estimate volume of piece three is 261.8 + 5,500 + 15,000 = 20,761.8mm^3

B. Calculations of actual volumes (with absolute and percentage errors)

Piece Number ONE: As the formula for the volume of a cube is $V= L^3$, and the formula for the volume of a cylinder is $V=\pi r^2 h$...

Total volume of chocolate piece no.1 = 70, 283.19mm^3

(See appendices 1 for workings)

Piece Number TWO: As the formula for the volume of a sphere is $V= 4/3\pi r^3$, the formula for the volume of a square-based pyramid is $V=a^2h/3$ and the formula for the volume of a rectangular prism is V= lwh...

Total volume of chocolate piece no.2 = 16,398.78mm^3

(See appendices 2 for workings)

Piece Number THREE: As the formula for the volume of a rectangular prism is V= lwh, the formula for the volume of a trapezoid is w[1/2h(b1+b2)] and formula for the volume of a semi-sphere is $\dfrac{\left(\frac{4}{3}\pi r^3\right)}{2}$ ...

Total volume of chocolate piece no.3 = 18,261.8mm^3

(See appendices 3 for workings)

C. Calculating Absolute and Percentage errors

NOTE: When calculating absolute and percentage errors, the following formulae were used...

VE = actual/exact value
VA= approximate value
Absolute error = VA-VE
Percentage error = [(VA-VE) ÷ VE] x100

Piece One:

Absolute Error = 99,077.16 - 70, 283.19 = 28793.97

Percentage Error = (28793.97 ÷ 70, 283.19) x 100 = 40.97%

Piece Two:

Absolute Error = 21,397.45 - 16,398.78 = 4998.67

Percentage Error = (4998.67 ÷ 16,398.78) x 100 = 30.48%

Piece Three:

Absolute Error = 20,761.8 - 18,261.8 = 2500

Percentage Error = (2500 ÷ 18,261.8) x 100 = 13.69%

According to these results, the predictions were mostly not accurate. However, there is a wide range of percentage errors – ranging from approx. 40% - 13%. Ways to make a more accurate prediction includes being more precise with measurements and making sure that they are achievable and realistic. If a business were to utilise the estimate values on production of this chocolate box, it would increase wastage significantly and would higher production cost.

Part Three: Calculating Surface Area

For the plastic tray, the total surface area = walls of box + base of box + walls of 3 chocolate 'slots'/compartments.

SA of walls of box + base of box = 25,250mm^2 ... (See appendices 4 for workings)

mistake made, Trivial error as student has shown correct working in appendix

SA of compartment one = 7,200mm^2 ... (See appendices 5 for workings)

SA of compartment two = 4,200mm^2 ... (See appendices 6 for workings)

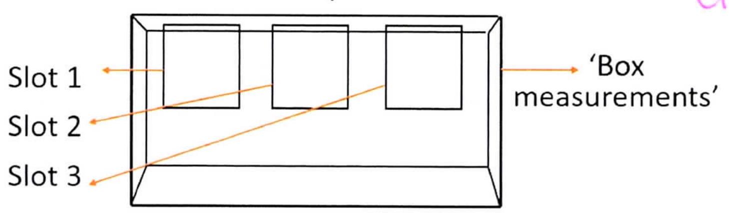
SA of compartment three = 3,750mm^2 ... (See appendices 7 for workings)

Therefore, total surface area of plastic tray = 25,250 + 7,200 + 4,200 + 3,750 = **40,400mm^2**

A

Good interpretation

Diagram of plastic tray



Part Four: Calculating Cost

(See appendices 8 for workings)

The total cost of the mini-box of chocolates = cost of chocolate + cost of tray material + cost of cardboard packaging, therefore;

As the cost of chocolate per kg is \$20.50 and the total weight of chocolate used to fill the designs is 0.10494kg...
The total cost of the chocolate is ≈ \$2.15

As the cost of the tray material is \$4.30 per m^2 and the total surface area of the tray is 0.0404m^2...
The total cost of the tray is ≈ \$0.15

As the cost of the cardboard packaging is \$2.80 per m^2 and the total SA of the box is 0.0522m^2...
The total cost of the cardboard packaging is ≈ \$0.15

In total, the cost of one mini-box of chocolates is ≈ \$2.45

good interpretation & conclusion

Report:

To summarise the results and measurements from all parts of this investigation, view the summary table below:

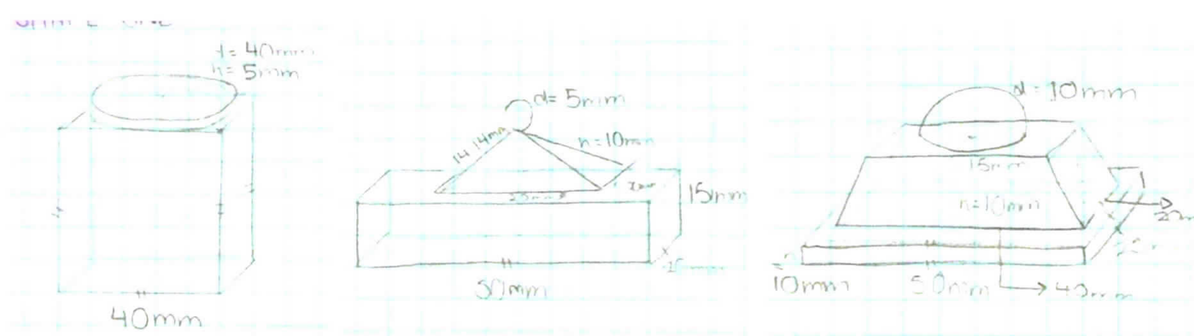
LABLE	Piece One	Piece Two	Piece Three	Plastic Tray	Packaging
Total Volume	70, 283.19mm^3	16,398.78mm^3	18,261.8mm^3	NA	NA
Estimate Volume	99,077.16mm^3	21,397.45mm^3	20,761.8mm^3	NA	NA
Total SA	NA	NA	NA	40,400mm^2	52,200mm^2
Absolute Error	28793.97	4998.67	2500	NA	NA
Percentage Error	40.97%	30.48%	13.69%	NA	NA
Cost	$2.15			$0.15	$0.15

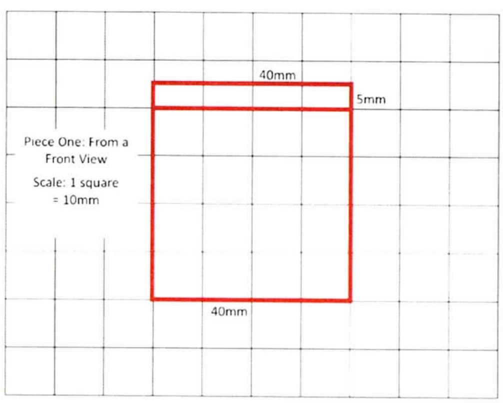
Great table to summarise

Although there were not many issues with this assignment, the only difficulty was the tray as multiple attempts were made to calculate the total surface area. One unresolved issue with this investigation is that it states that there was a 1kg bag of chocolate for sale at a price of $20.50. If a business were to produce this mini-chocolate box, they would have to buy more than a percentage of a bag if wanting to mass-produce. Limitations of this assignment include the restriction on the shapes used (e.g. Nothing more than simple shapes put together to create 'complex' shapes). Assumptions were also made in this assignment, for example, the ability to obtain resources such as cardboard packaging and/or a special tray material. Another assumption was made, suggesting that all the chocolates had to be solid instead of hollow.

same would apply for plastic & packaging

Figures below show the original 3D sketches from part one as well as a scaled diagram of piece one.





Appendix:

Figure 1:

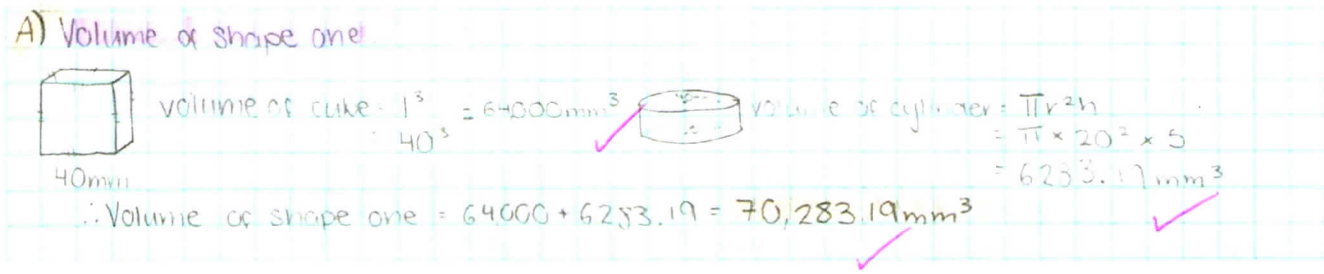


Figure 2:

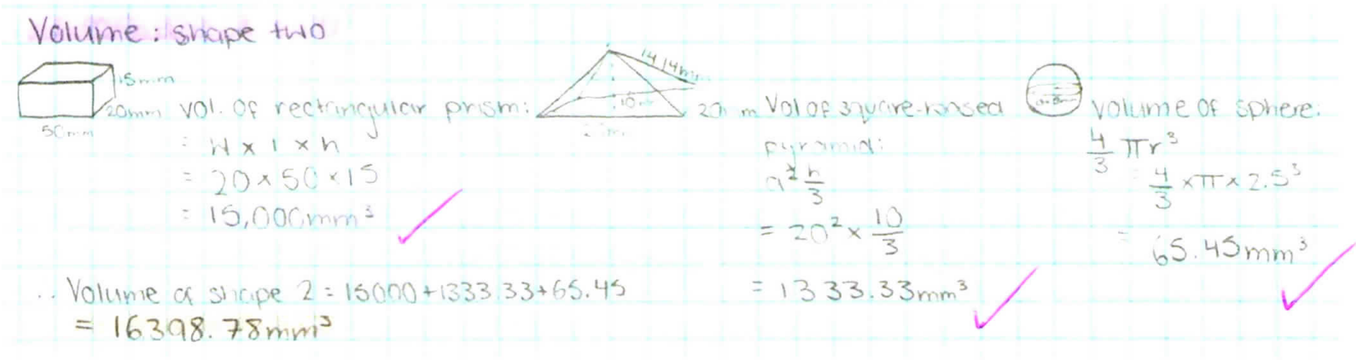


Figure 3:

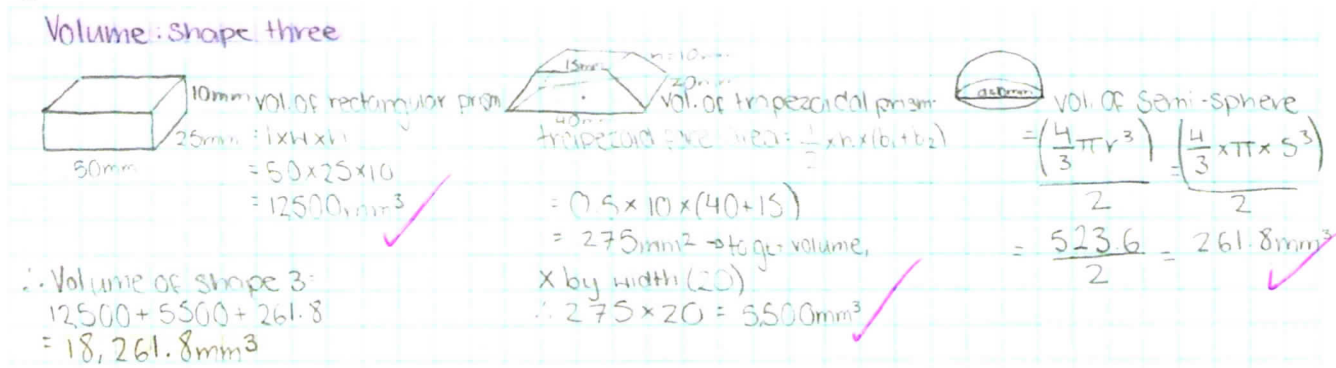


Figure 4:

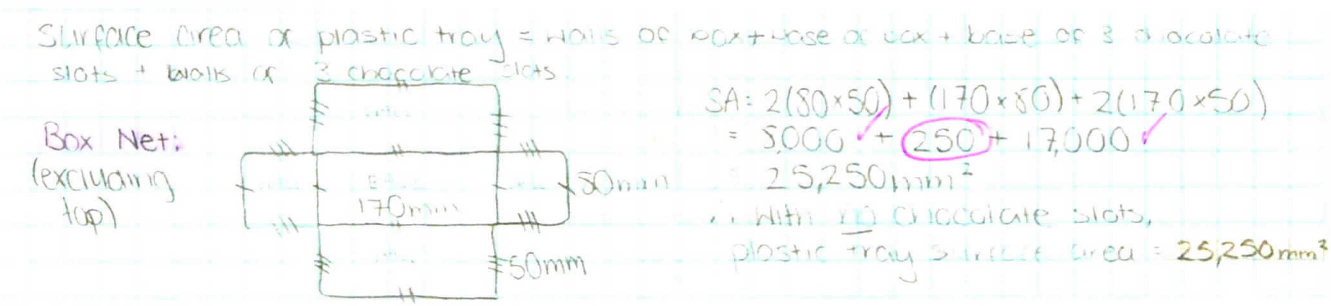
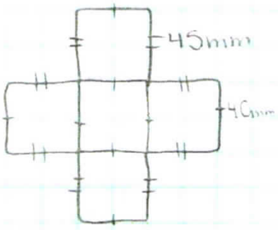


Figure 5:

Shape one divit: (NOTE: ALL NET DIAGRAMS EXCLUDE TOP)

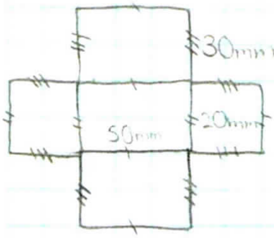


∴ SA of rectangular prism (excluding top)
$4(45 \times 40) + (40 \times 40)$
$= 5800mm^2$

As a "base" is already included in tray...
$5800 - (40 \times 40)$
$= 7{,}200mm^2$

Figure 6:

Shape 2 divit:

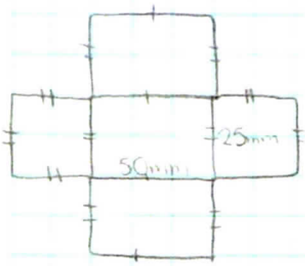


∴ SA of rectangular prism: $2(wl + hl + hw)$
$= 2(20 \times 50) + 2(30 \times 50) + 2(30 \times 20)$
$= 6{,}200mm^2$
Excluding top face,
$6{,}200 - (20 \times 50)$
$= 5{,}200mm^2$

" "
$5{,}200 - (50 \times 20)$
$= 4{,}200mm^2$

Figure 7:

Shape 3 divit:



SA: $2(wl + hl + hw)$
$= 2(25 \times 50) + 2(25 \times 50) + 2(25 \times 25)$
$= 6250mm^2$
Excluding top face,
$= 6250 - (50 \times 25)$
$= 5000mm^2$

" "
$5000 - (50 \times 25)$
$= 3750mm^2$

Figure 8:

PART FOUR

cost of chocolate: \$20.50 per kg
total amount of choc. used in all shapes...
$70{,}253.79 + 16{,}398.75 + 15{,}261.8$
$= 104943.77mm^3$
$= 104.94cm^3 = 104.94g$
$= 0.10494kg \times 20.50$
$= \$2.15$
∴ 2.15 for chocolate

cost of tray: \$4.30 per m^2.
total tray area:
$40{,}400mm^2$
$= 0.0404m^2$
∴ 0.0404×4.30
$= \$0.17372$
∴ ≈ \$0.15 for tray

total SA of packaging:
$2(50 \times 50) + 2(170 \times 50) + 2(170 \times 50)$
$= 52200mm^2$
cost of packaging per m^2: \$2.80
∴ $52200mm^2$
$= 0.0522m^2$
0.0522×2.80
$= 0.1461$
∴ ≈ \$0.15 for packaging

∴ Total cost $= \$2.15 + 0.15 + 0.15$
$= \$2.45$