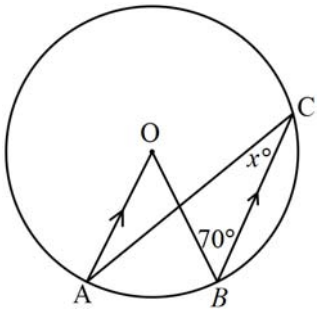
QUESTION 1

Find x in the following diagrams giving reasons.

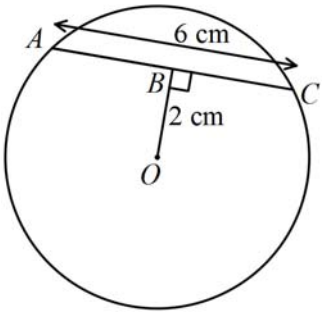
(a)



(2 marks)

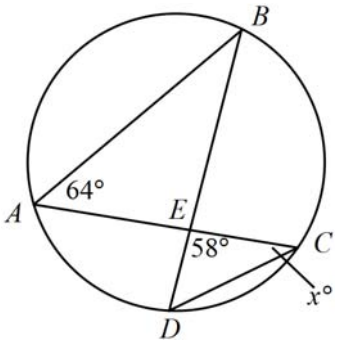
(b)

Find the radius



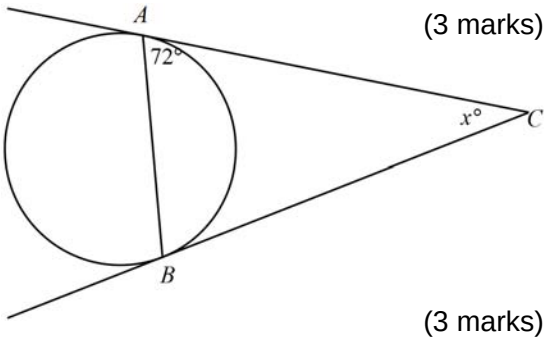
(2 marks)

(c)



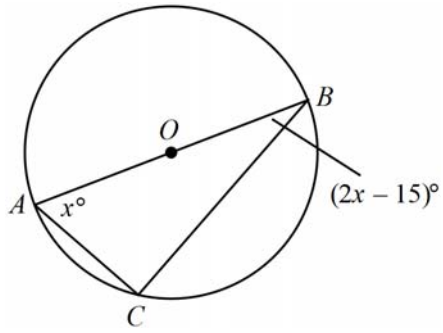
(3 marks)

(d)



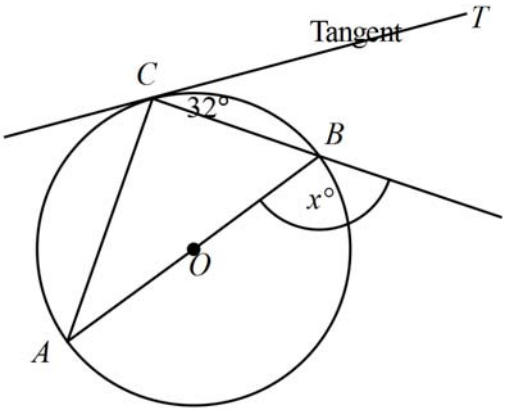
(3 marks)

(e)



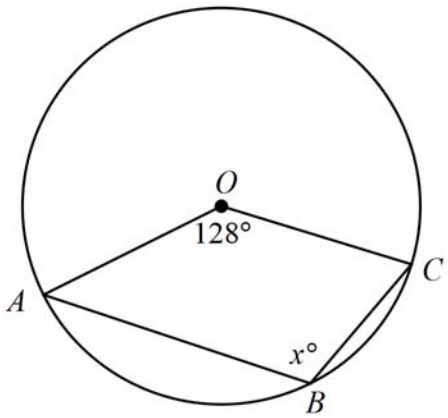
(3 marks)

(f)



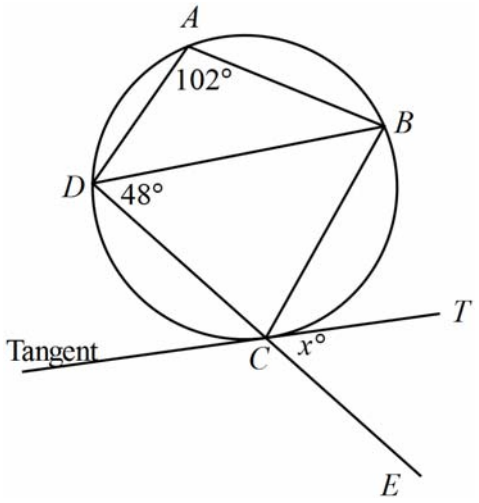
(3 marks)

(g)



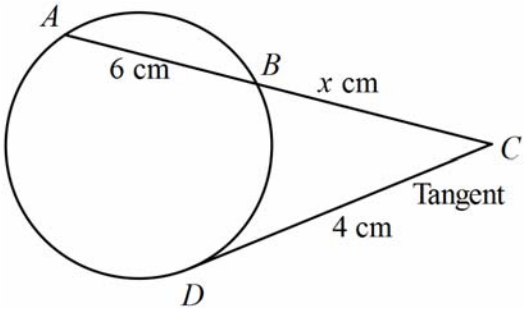
(2 marks)

(h)



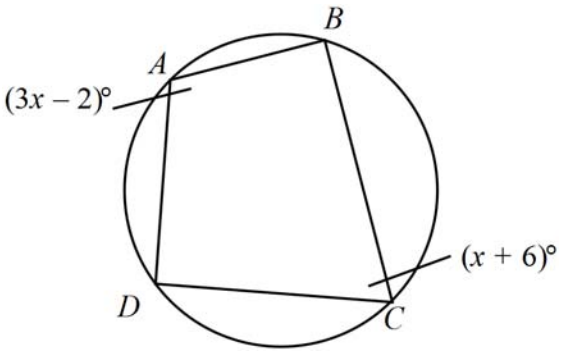
(4 marks)

(i)



(3 marks)

(j)

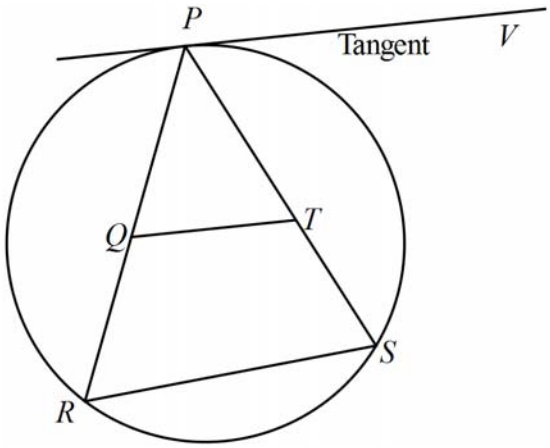


(2 marks)

QUESTION 2

PV is a tangent to the circle and QRST is a cyclic quadrilateral.

Prove that PV is parallel to QT.

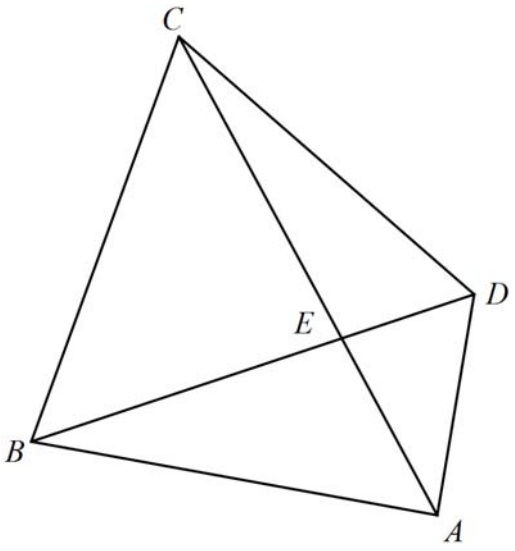


(4 marks)

QUESTION 3

ABCD is a quadrilateral. Diagonals AC and BD intersect at E.

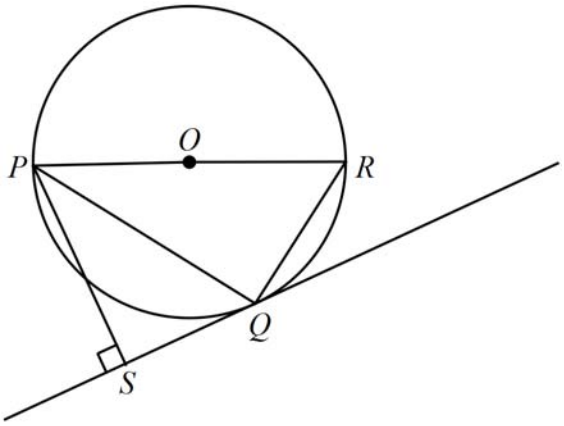
If AC bisects $\angle BAD$ and $\angle ABC = \angle AED$, prove that ABCD is a cyclic quadrilateral.



(3 marks)

QUESTION 4

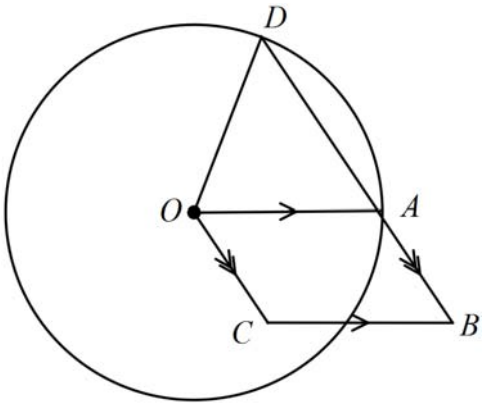
Triangle PQR is inscribed in a circle with PR as a diameter. The perpendicular from P to the tangent at Q meets the tangent at S, prove that PQ bisects $\angle SPR$.



(4 marks)

QUESTION 5

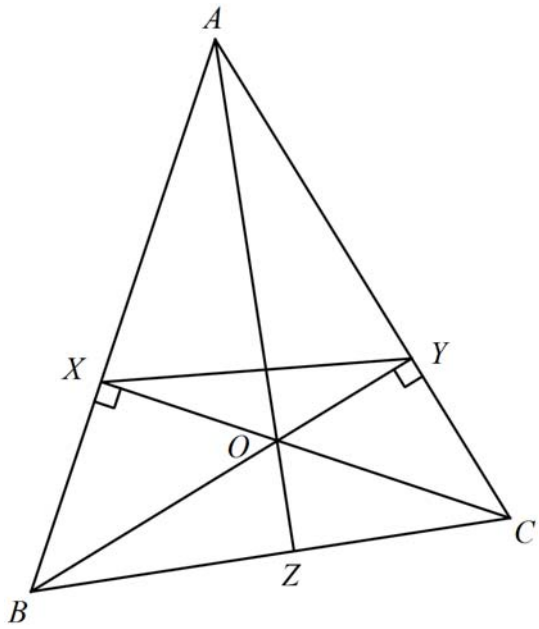
OABC is a parallelogram. A circle, centre at O and radius OA is drawn. BA produced meets the circle at D. Prove that DOCB is a cyclic quadrilateral.



(5 marks)

QUESTION 6

Triangle ABC has perpendiculars CX and BY as shown.



(a) What can be said about quadrilaterals AXOY and BXYC? Give reasons.

(2 marks)

(b) Prove that $\angle XAO = \angle XYO = \angle XCB$.

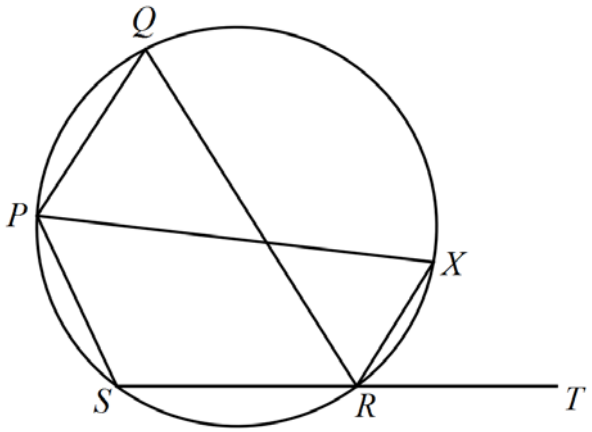
(2 marks)

(c) Prove that AZ is perpendicular to BC.

(2 marks)

QUESTION 7

RX is the bisector of $\angle QRT$. Prove that PX bisects $\angle QPS$.

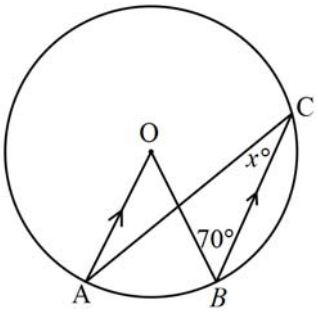


(3 marks)

QUESTION 1

Find x in the following diagrams giving reasons.

(a)

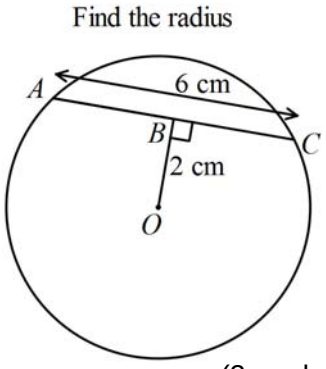


$\angle AOB = 70°$ (Alternate angles =) ✓

$\therefore \quad x = 35$ (Angle at the centre theorem) ✓

(2 marks)

(b)

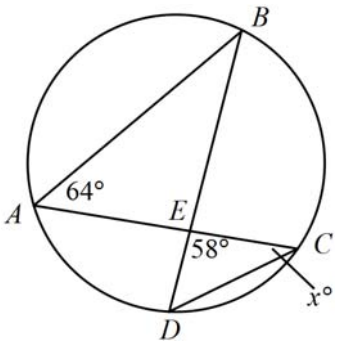


$BC = 3$ (Chord perpendicular to radius bisects it) ✓

$\therefore \quad r = \sqrt{4+9} = \sqrt{13}$ ✓

(2 marks)

(c)



$\angle BAE = \angle BDC = 64$ (Angles in same segment) ✓

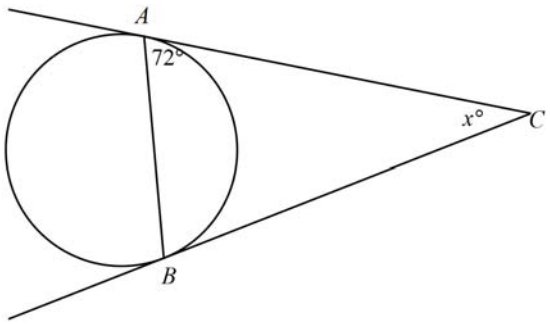
$\therefore \quad x = 180 - (58 + 64)$ (Triangle = 180) ✓

$\therefore \quad x = 58$ ✓

(3 marks)

(d) $\angle ABC = 72$ (Tangents from ext point theorem & Isos Triangle) ✓ ✓

$\therefore \quad x = 180 - 2 \times 72 = 36$ (Triangle = 180) ✓



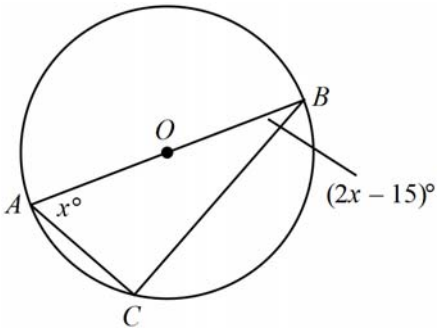
(3 marks)

(e) $\angle ACB = 90$ (Angle in semi-circle) ✓

$\therefore x + (2x - 15) = 90$ (Triangle =180) ✓

$\therefore \quad 3x = 105$

$\therefore \quad x = 35$ ✓

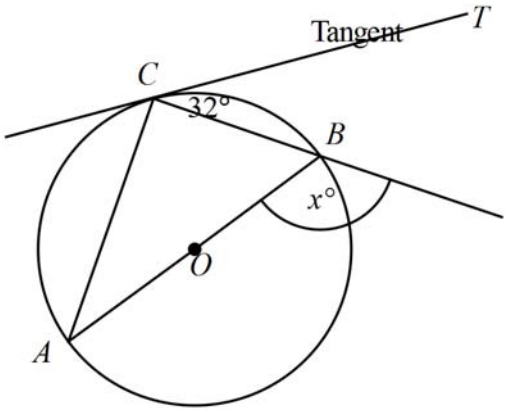


(3 marks)

(f) $\angle ACB = 90$ (Angle in semi-circle) ✓

$\angle CAB = 32$ (Angle between tan and chord) ✓

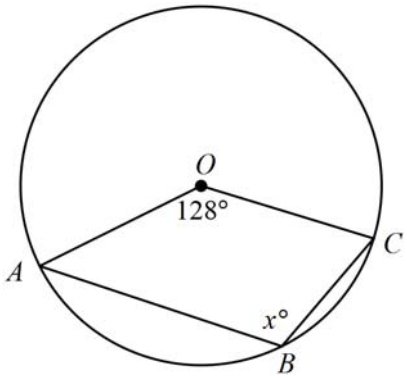
$\therefore \quad x = 90 + 32 = 112$ (Exterior angle of triangle) ✓



(3 marks)

(g) $\angle AOC$ (reflex) = 360 – 128 = 232 (Circle = 360) ✓

$\therefore x = 116$ (Angle at centre) ✓



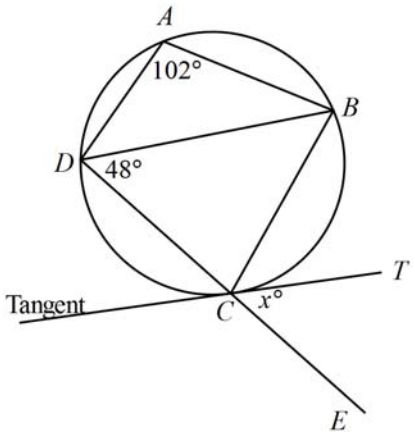
(2 marks)

(h) $\angle DCB = 78$ (Opp angles cyclic quad supp) ✓

$\therefore \quad \angle BCT = 48$ (Angle between tangent and chord) ✓

$\therefore x + 48 + 78 = 180$ (Straight line =180) ✓

$\therefore \quad x = 54$ ✓



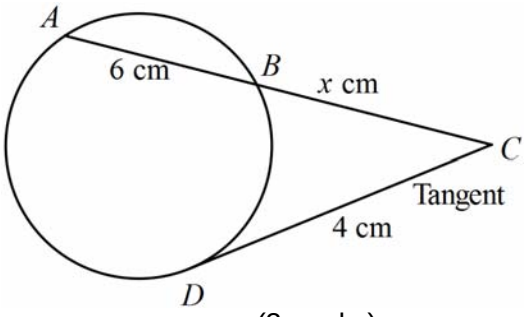
(4 marks)

(i) $x(x+6) = 4^2$ (Sec-Tan theorem) ✓

$\therefore \quad x^2 + 6x - 16 = 0$

$\therefore \quad (x+8)(x-2) = 0$ ✓

$\therefore \quad x = -8$ or 2 but $x > 0 \ \therefore x = 2$ ✓

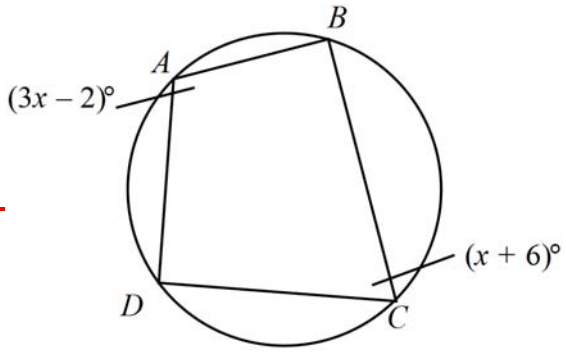


(3 marks)

(j) $(3x-2) + (x+6) = 180$ (Opp angles supp) ✓

$\therefore \quad 4x + 4 = 180$

$\therefore \quad x = 44$ ✓

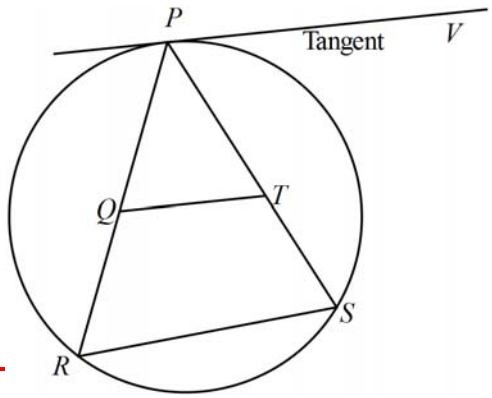


(2 marks)

QUESTION 2

PV is a tangent to the circle and QRST is a cyclic quadrilateral.

Prove that PV is parallel to QT.



Let $\angle PRS = \alpha$

$\therefore \quad \angle QTS = 180 - \alpha$ (Opp angles cyclic quad) ✓

$\angle SPV = \alpha$ (Angles between tan and chord) ✓

$\therefore \quad \angle PTQ = \alpha$ (Straight line = 180) ✓

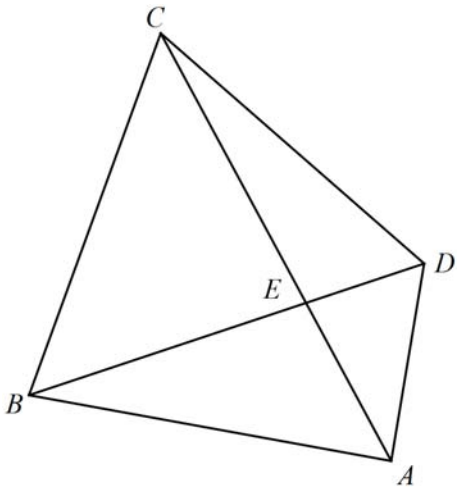
$\therefore \quad \angle VPT = \angle PTQ = \alpha$ (Alternate angles) ✓

(4 marks)

QUESTION 3

ABCD is a quadrilateral. Diagonals AC and BD intersect at E.

If AC bisects $\angle BAD$ and $\angle ABC = \angle AED$, prove that ABCD is a cyclic quadrilateral.



Let $\angle BAC = \angle CAD = \alpha$ (Given bisector)

Let $\angle ABC = \angle AED = \beta$ (Given bisector)

In ΔABC $\angle BCA = 180 - (\alpha + \beta)$ (Triangle = 180) ✓

In ΔEDA $\angle EDA = 180 - (\alpha + \beta)$ (Triangle = 180) ✓

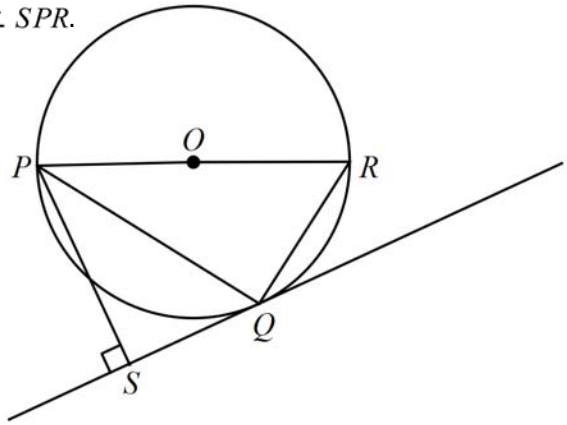
$\therefore \quad \angle BCA = \angle BDA = 180 - (\alpha + \beta)$

$\Rightarrow$ AB subtends = $\angle$ at C and D $\therefore$ Cyclic Quad ✓

(3 marks)

QUESTION 4

Triangle PQR is inscribed in a circle with PR as a diameter. The perpendicular from P to the tangent at Q meets the tangent at S, prove that PQ bisects $\angle SPR$.



Let $\angle QPR = \alpha$

$\therefore \quad \angle PQR = 90$ (Angle in semi-circle) ✓

$\therefore \quad \angle PRQ = 90 - \alpha$ (Triangle = 180) ✓

$\therefore \quad \angle PQS = 90 - \alpha$ (Angle between tan and chord) ✓

$\therefore \quad \angle SPQ = \alpha$ (Triangle = 180) ✓

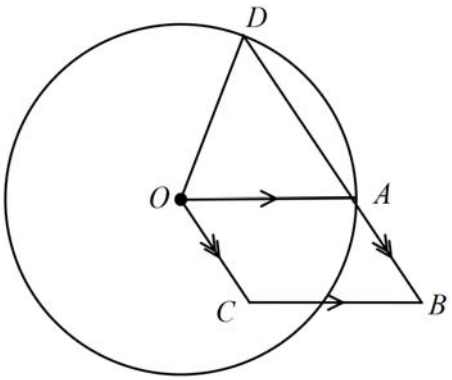
$\Rightarrow$ PQ bisects $\angle SPR$

(4 marks)

QUESTION 5

OABC is a parallelogram. A circle, centre at O and radius OA is drawn. BA produced meets the circle at D.

Prove that DOCB is a cyclic quadrilateral.



Let $\angle ABC = \alpha$

$\therefore \quad \angle COA = \alpha$ (Opp angles parallelogram) ✓

$\therefore \quad \angle OAD = \alpha$ (Corr angles =) ✓

$\therefore \quad \angle ODA = \alpha$ (Isos triangle radii) ✓

$\therefore \quad \angle AOD = 180 - 2\alpha$ (Triangle = 180) ✓

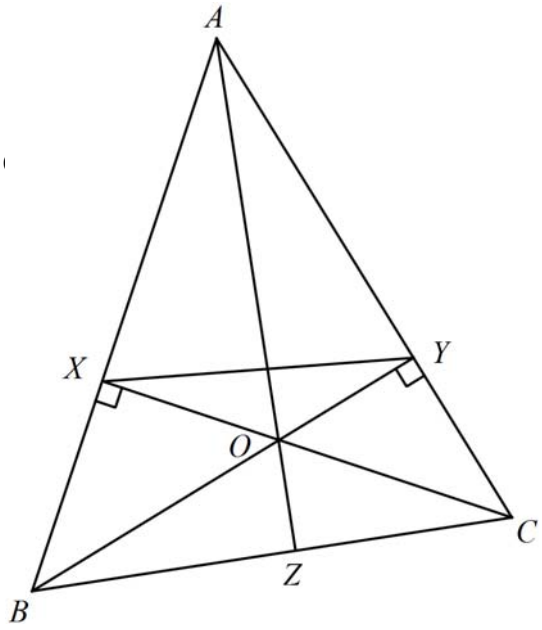
$\therefore \quad \angle COD = (180 - 2\alpha) + \alpha = 180 - \alpha$

$\Rightarrow \quad \angle DOC + \angle DBC = (180 - \alpha) + \alpha = 180$ (Opp angles cyclic quad) ✓

(5 marks)

QUESTION 6

Triangle ABC has perpendiculars CX and BY as shown.



(a) What can be said about quadrilaterals AXOY and BXYC?

$AXOY$ cyclic quad $\angle AXO + \angle AYO = 90 + 90 = 180$ ✓

$\angle BXC = \angle BYC = 90 \Rightarrow BXYC$ cyclic quad ✓

(2 marks)

(b) Prove that $\angle XAO = \angle XYO = \angle XCB$.

$\angle XAO = \angle XYO$ (XAOY cyclic quad) ✓

$\angle XYO = \angle XYB = \angle XCB$ (Cyclic quad) ✓

(2 marks)

(c) Prove that AZ is perpendicular to BC.

$\angle XAO = \angle XAZ$ and $\angle XCB = \angle XCZ$

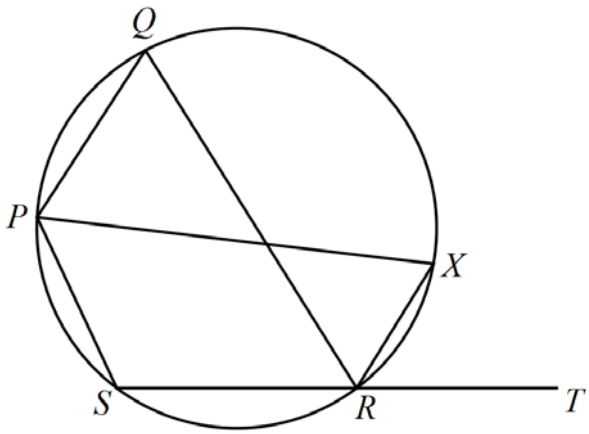
$\Rightarrow \angle XAZ = \angle XCZ \Rightarrow$ Cyclic quad $XACZ$ ✓

$\therefore \angle AXC = \angle AZC = 90 \Rightarrow AZ \perp BC$ ✓

(2 marks)

QUESTION 7

RX is the bisector of $\angle QRT$. Prove that PX bisects $\angle$ QPS.



Let $\angle QRX = \angle XRT = \alpha$ (*Given*)

$\angle QPX = \angle QRX = \alpha$ (Angles same segment)

Since $PXRS$ cyclic quad

$\Rightarrow \angle SPX = \alpha$ (Exterior ang cyclic quad) ✓

$\therefore \angle QPX = \angle SPX = \alpha \Rightarrow$ Bisected ✓

(3 marks)